Arabic nominals in HPSG: A verbal noun perspective

Md. Sadiqul Islam
Bangladesh University of Engineering and Technology, Dhaka

Mahmudul Hasan Masum
Bangladesh University of Engineering and Technology, Dhaka

Md. Shariful Islam Bhuyan
Bangladesh University of Engineering and Technology, Dhaka

Reaz Ahmed
Bangladesh University of Engineering and Technology, Dhaka

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Abstract

Semitic languages exhibit rich nonconcatenative morphological operations, which can generate a myriad of derived lexemes. Especially, the feature rich, root-driven morphology in the Arabic language demonstrates the construction of several verb-derived nominals (verbal nouns) such as gerunds, active participles, passive participles, locative participles, etc. Although HPSG is a successful syntactic theory, it lacks the representation of complex nonconcatenative morphology. In this paper, we propose a novel HPSG representation for Arabic nominals and various verb-derived nouns. We also present the lexical type hierarchy and derivational rules for generating these verb-derived nominals using the HPSG framework.

1 Introduction

HPSG analyses for nonconcatenative morphology in general and for Semitic (Arabic, Hebrew and others) languages in particular are relatively new (Bhuyan and Ahmed, 2008b; Mutawa et al., 2008; Kihm, 2006; Bhuyan and Ahmed, 2008c; Riehemann, 2000; Bird and Klein, 1994; Bhuyan and Ahmed, 2008a; Islam et al., 2009). However, the intricate nature of Arabic morphology motivated several research projects addressing the issues (Beesley, 2001; Buckwalter, 2004; Smrž, 2007). HPSG representations of Arabic verbs and morphologically complex predicates are discussed in (Bhuyan and Ahmed, 2008b,a,c). An in-depth analysis of declensions in Arabic nouns has been presented in (Islam et al., 2009). The diversity and importance of Arabic nominals is broader than that of their counterparts in other languages. Modifiers, such as adjectives and adverbs, are treated as nominals in Arabic. Moreover, Arabic nouns can be derived from verbs or other nouns. Derivation from verbs is one of the primary means of forming Arabic nouns, for which no HPSG analysis has been conducted yet.

Arabic noun can be categorized based on several dimensions. Based on derivation, Arabic nouns can be divided into two categories as follows:

1. Non-derived nouns: These are not derived from any other noun or verb.
2. Derived nouns: These are derived from other nouns or verbs.

An example of a non-derived, static noun is جَحْسَانُ (hiṣānūn - which means “horse”): it is not derived from any noun or verb and no verb is generated from this word. On the other hand, كَاتِبٌ (katību - which means “writer”) is an example of

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a derived noun. This word is generated from the verb كَتَبَ (kataba) which means “He wrote” in English. This simple example provides a glimpse of the complexity of the derivational, nonconcatenative morphology for constructing a noun from a verb in Arabic. In this paper, we analyze and propose the HPSG constructs required for capturing the syntactic and semantic effects of this rich morphology.

An HPSG formalization of Arabic nominal sentences has been presented in (Mutawa et al., 2008). The formalization covers seven types of simple Arabic nominal sentences while taking care of the agreement aspect. In (Kihm, 2006), an HPSG analysis of broken plural and gerund has been presented. Main assumption in that work evolves around the Concrete Lexical Representations (CLRs) located between an HPSG type lexicon and phonological realization. But in that work the authors have not addressed other forms of verbal nouns including participles. Our contributions to an HPSG analysis of Arabic nouns presented in this paper are as follows:

- We capture the syntactic and semantic effects of Arabic morphology in Section 3.1.
- In Section 3.1 we formulate the structure of attribute value matrix (AVM) for Arabic noun.
- We indicate the location of verb-derived nouns in the lexical type hierarchy in Section 3.2.
- We extend the basic AVM of nouns for verbal nouns (Section 3.3).
- We propose lexical construction rules for the derivation of verbal nouns from verbs in Section 3.3.

2 Verb Derived Noun in Arabic Grammar

2.1 Arabic morphology

Arabic verb is an excellent example of nonconcatenative root-pattern based morphology. A combination of root letters are plugged in a variety of morphological patterns with priorly fixed letters and particular vowel melody that generates verbs of a particular type which has some syntactic and semantic information (Bhuyan and Ahmed, 2008b). Figure 1 shows how different sets of root letters plugged into a vowel pattern generate different verbs with some common semantic meanings.

Besides vowel pattern, a particular verb type depends on the root class\(^1\) and verb stem. This root class is determined on basis of the phonological characteristics of the root letters. Root classes can be categorized on basis of the number of root letters, position or existence of vowels among these root letters and the existence of a gemination (tashdeed). Most Arabic verbs are generated from triliteral

\(^1\)We call a set of roots, which share a common derivational and inflectional paradigm, a root class.
and quadrilateral roots. In Modern Standard Arabic five character root letters are obsolete. Phonological and morphophonemic rules can be applied to various kinds of sound and irregular roots. Among these root classes, sound root class is the simplest and it is easy to categorize its morphological information. A sound root consists of three consonants all of which are different (Ryding, 2005). On the other hand, non-sound root classes are categorized in several subtypes depending on the position of weak letters (i.e., vowels) and gemination or hamza. All these subtypes carry morphological information.

From any particular sequence of root letters (i.e., triliteral and quadriliteral), up to fifteen different verb stems may be derived, each with its own template. These stems have different semantic information. Western scholars usually refer to these forms as Form I, II, . . . , XV. Form XI to Form XV are rare in Classical Arabic and are even more rare in Modern Standard Arabic. These forms are discussed in detail in (Ryding, 2005). Here we give examples of each of the well-known ten verb forms.

1. Form I (Transitive): *kataba*  (كَتَبَ) – “He wrote”.

2. Form II (Causative): *kattaba*  (كَتَبَ) – “He caused to write”.

Figure 1: Root-pattern morphology: 3rd person singular masculine sound perfect active form-I verb formation from same pattern
3. Form III (Ditransitive): \(kātāba\) (كانَب) – “He corresponded”.

4. Form IV (Factitive): \(aktaba\) (أَكَتَب) – “He dictated”.

5. Form V (Reflexive): \(takattaba\) (تَكَتَّب) – “It was written on its own”.

6. Form VI (Reciprocity): \(takātāba\) (تُكَتَّب) – “They wrote to each other”.

7. Form VII (Submissive): \(inkataba\) (إِن‌َكَتَب) – “He was subscribed”.

8. Form VIII (Reciprocity): \(iktataba\) (إِكْتَتَب) – “They wrote to each other”.

9. Form IX (Color or bodily defect): \(ihmarra\) (إِحْمَرَر) – “It turned to red”.

10. Form X (Control): \(istaktaba\) (إِسْتَكَتَب) – “He asked to write”.

It is worth mentioning that Form I has eight subtypes depending on the vowel following the middle letter in perfect and imperfect forms. Some types of verbal noun formation depend on these subtypes. Any combination of root letters for Form I verb will follow any one of these eight patterns. We refer these patterns as Form IA, IB, IC, . . . , IH. These subtypes are shown in Table 1 with corresponding examples. For example, the vowels on the middle letter for Form IA: nasara yansuru are \(a\) and \(u\) for perfect and imperfect forms, respectively. Similarly, other forms depend on the combination of vowels on these two positions. Not all kinds of combinations exist. In Form IH, the middle letter is a long vowel and there is no short vowel on this letter. No verbal noun is derived from Form IH subtype.

In summary, we can generate different types of verbal nouns based on these verb forms, root classes and vowel patterns.

<table>
<thead>
<tr>
<th>Form</th>
<th>Example</th>
<th>Perfect mid-vowel</th>
<th>Imperfect mid-vowel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Form IA</td>
<td>nasara yansuru</td>
<td>(a)</td>
<td>(u)</td>
</tr>
<tr>
<td>Form IB</td>
<td>daraba yadribu</td>
<td>(a)</td>
<td>(i)</td>
</tr>
<tr>
<td>Form IC</td>
<td>fataha yafahu</td>
<td>(a)</td>
<td>(a)</td>
</tr>
<tr>
<td>Form ID</td>
<td>samira yasmara</td>
<td>(i)</td>
<td>(a)</td>
</tr>
<tr>
<td>Form IE</td>
<td>karuma yakrumu</td>
<td>(u)</td>
<td>(u)</td>
</tr>
<tr>
<td>Form IF</td>
<td>hasiba yahsibu</td>
<td>(i)</td>
<td>(i)</td>
</tr>
<tr>
<td>Form IG</td>
<td>fadula yafidlu</td>
<td>(u)</td>
<td>(i)</td>
</tr>
<tr>
<td>Form IH</td>
<td>kāda yakādu</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2.2 The classification of verbal nouns

In this section we discuss the eight types of nouns derived from verbs (LearnArabicOnline.com, 2003-2010a):

1. Gerund (اسم مصدر) - *isms madar* - names the action denoted by its corresponding verb.

2. Active participle (اسم الفاعل) - *isms alf'il* - entity that enacts the base meaning i.e. the general actor.

3. Hyperbolic participle (اسم المبالغة) - *isms almubalagah* - entity that enacts the base meaning exaggeratedly. So it modifies the actor with the meaning that actor does it excessively.

4. Passive participle (اسم المفعول) - *isms almuw'ul* - entity upon which the base meaning is enacted. Corresponds to the object of the verb.

5. Resembling participle (اسم الفاعل المتقئ) - *isms alifatulmu'sabahah* - entity enacting (or upon which is enacted) the base meaning intrinsically or inherently. Modifies the actor with the meaning that the actor does the action inherently.

6. Utilitarian noun (اسم الألة) - *isms alalah* - entity used to enact the base meaning i.e. instrument used to conduct the action.

7. Locative noun (اسم الظروف) - *isms al'zaf* - time or place at which the base meaning is enacted.

8. Comparative and superlative (اسم التفاسيل) - *isms altafdil* - entity that enacts (or upon whom is enacted) the base meaning the most. In Arabic, this type of word is categorized as a noun, but it is similar to an English adjective.

Examples of these eight types of verbal nouns are presented in Table 2. Each of these types can be subcategorized on the basis of types of verbs. To understand complete variation of verb and its morphology we should have some preliminary knowledge of the Arabic verb.

3 HPSG Formalism for Verbal Noun

In this section we model the categories of verbal nouns and their derivation from different types of verbs through HPSG formalism. We adopt the SBCG version of HPSG (Sag, 2010) for this analysis. We discuss different HPSG types of root verbs and verbal nouns and then propose a multiple inheritance hierarchical model for Arabic verbal nouns. We give an AVM for nouns and extend it for verbal nouns then propose how to get a sort description of an AVM for verbal nouns from the type hierarchy. Finally, we propose construction rules of verbal nouns from root verbs.
Table 2: Different types of verbal nouns.

<table>
<thead>
<tr>
<th>Source verb</th>
<th>Verb derived noun</th>
<th>Example</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gerund</td>
<td>أَلْيَمَ</td>
<td>&quot;Knowing&quot;</td>
<td>(al’ilmu)</td>
</tr>
<tr>
<td>Active participle</td>
<td>أَلْيَمَ</td>
<td>&quot;One who knows&quot;</td>
<td>(alimun)</td>
</tr>
<tr>
<td>Hyperbolic participle</td>
<td>أَلْيَمَ</td>
<td>&quot;One who knows a lot&quot;</td>
<td>(allamun)</td>
</tr>
<tr>
<td>Passive participle</td>
<td>مَلْعَمَ</td>
<td>&quot;That which is known&quot;</td>
<td>(malumun)</td>
</tr>
<tr>
<td>Resembling participle</td>
<td>أَلْيَمَ</td>
<td>&quot;One who knows intrinsically&quot;</td>
<td>(aliymun)</td>
</tr>
<tr>
<td>Utilitarian noun</td>
<td>مَلْعَمَ</td>
<td>&quot;Through which we know&quot;</td>
<td>(milamun)</td>
</tr>
<tr>
<td>Locative noun</td>
<td>مَلْعَمَ</td>
<td>&quot;Where/when we know&quot;</td>
<td>(malimin)</td>
</tr>
<tr>
<td>Comparative and Superlative</td>
<td>أَلْيَمَ</td>
<td>&quot;One who knows the most&quot;</td>
<td>(lamu)</td>
</tr>
</tbody>
</table>

3.1 AVM of Arabic nouns

We modify the SBCG feature geometry for English and adopt it for Arabic. The SBCG AVMs for nouns in English and Arabic are shown in Figure 2 and Figure 3, respectively.

The PHON feature is out of the scope of this paper. The MORPH feature captures the morphological information of signs and replaces the FORM feature of English AVMs. The value of the feature FORM is a sequence of morphological objects (formatives); these are the elements that will be phonologically realized within the sign’s PHON value (Sag, 2010). On the other hand, MORPH is a function feature. It not only contains these phonologically realized elements but also contains their origins. MORPH contains two features - ROOT and STEM. ROOT feature contains root letters for the following cases:

1. The root is characterized as a part of a lexeme, and is common to a set of derived or inflected forms
2. The root cannot be further analyzed into meaningful units when all affixes are removed
Figure 2: SBCG noun AVM for English

Figure 3: SBCG noun AVM for Arabic
3. The root carries the principal portion of meaning of the lexeme

In rest of the cases, the content of this feature is empty.

The STEM feature contains a list of letters, which comprise the word or phrase or lexeme. We can identify any pattern in the lexeme by substituting the root letters to the placeholders in STEM. As an example, the ROOT of the lexeme ‘kataba’ contains ‘k’, ‘t’ and ‘b’ and the pattern of the STEM is \( \_\_a.\omega_\omega.a \). Without the existence of this pattern, the ROOT is irrelevant. Thus a pattern bears the syntactic information and a ROOT bears the semantic information. Lexemes which share a common pattern must also share some common syntactic information. Similarly, lexemes which share a common root must also share some common semantic information. STEM is derived from the root letters by nonconcatenative morphology.

The SYN feature contains CAT, VAL and MRKG features. We modify the CAT feature of SBCG to adopt it for Arabic language. Note that, for all kinds of verbal nouns the sort description of the CAT feature is noun. In Arabic there are only three parts of speech (POS) for lexemes or words: noun/pronoun, verb and particle. Any verbal noun serving as a modifier is also treated as noun. In that case, the list of FRAMES under SEM feature will contain the modifier-frame. In the case of the Arabic noun, the CAT feature consists of CASE, DEF, SELECT, XARG and LID features.

The DEF feature denotes the value of definiteness of an Arabic noun. There are eight ways by which a noun word or lexeme may be definite (LearnArabicOnline.com, 2003-2010b). Personal pronouns such as “he”, “I” and “you” are inherently definite. Proper nouns are also definite. \( \text{اللَّه} \) (al-lāhu) is another instance of definite lexeme. These examples confirm that definiteness must be specifiable at the lexeme level. The article al also expresses the definite state of a noun of any gender and number. Thus if the state of a noun is definite, the noun contains yes as the value of DEF, otherwise its value will be no. There is a significant role of this definiteness (DEF) feature in Arabic. A nouns and its modifier must agree on the DEF feature value. For example, \( \text{الْكِتَابُ} \text{'l-\text{أَهْمَارَ} \) means “the red book”’. \( \text{الْكِتَابُ} \) (alkitābu) means “the book” and \( \text{أَحْمَرُ} \) (ahmaru) means “red”. As “red” is used as a modifier for “the book”, the definiteness prefix al has been added to \( \text{أَحْمَرُ} \) yielding \( \text{الَّأَحْمَرُ} \).

The agreement features are PERSON, NUMBER, GENDER and HUM. These are contained inside the INDEX feature under SEM. The HUM feature denotes humanness. Depending on languages, agreement may have gender, human/non-human, animate/inanimate or shape features (Pollard and Sag, 1994). In Arabic, Humanness is a crucial grammatical factor for predicting certain kinds of plural formation and for the purpose of agreement with other components of a phrase or clause within a sentence. The grammatical criterion of humanness only applies to nouns in the plural form. As an example, “these boys are intelligent” (\( \text{هُدْوَّارَ} \text{أَلْوَّلَادُ} \text{رَدْكِيْعَاء} \) and “these dogs are intelligent” (\( \text{هُدْوَّرَ} \text{أَلْوَّلَادُ} \text{رَدْكِيْعَاء} \).}
hadīhi ‘ikilābu dakiyyatun). Both sentences are plural. But the former refers to human beings whereas latter refers to non-humans. So the same word “intelligent” (dakiyyun) has taken two different plural forms in two sentences: أُدِكْيِيْلْ (‘adkiyār) and دَكَيْيَةٌ (dakiyyatun). In the case of boys, it is in the third person masculine plural form (أُدِكْيِيْلْ - ‘adkiyār) whereas in case of dogs, it is in the third person feminine singular form (دَكَيْيَةٌ - dakiyyatun). If the noun refers to a human being then the value of HUM is yes, otherwise it is no. Thus, along with PERSON, NUMBER and GENDER, we keep HUM as an agreement feature.

The value of PERSON for Arabic noun can be 1st, 2nd or 3rd. There are three number values in Arabic. So, the value of NUMBER can be sg, dual or pl denoting singular, dual or plural, respectively. The GENDER feature contains either male or female value.

![Figure 4: Lexical type hierarchy of Arabic noun lexeme.](image)

### 3.2 Type Hierarchy of Verbal Noun

As mentioned in Section 2.1, the derivation of verbal nouns from verbs depends on the number of root letters, the verb form and the root type. In Figure 4 we give a type hierarchy of Arabic verbal nouns.

As shown in Figure 4, eight types of verbal nouns are immediate daughters
of *verb-derived-noun*. Each of these eight different verbal nouns can be subcategorized on the basis of the properties of the root verb, which are mentioned in Section 2.1. Each verb carries distinct information on these properties, which form the dimensions of classification for verbs. So, the three dimensions for root verbs are: number of root letter, type of the root and verb form. For lack of space we discuss in detail only the subtypes of active participles.

In Figure 5, *active-participle* is at the root. Categorizing it along the number of letter in root verb, we get two types of active participles, derived from triliteral and quadriliteral root verb. Some verbal nouns are generated from triliteral roots only. For example, comparative and superlative nouns are derived only from triliteral Form-I verbs. Also, verbal nouns derived from triliteral roots have known patterns. Again classifying the active participle along the root type, we find several types of roots and thus verbal nouns. Categorizing along the verb form dimension, we get Form-I, . . ., Form-X active participles. Categories in one dimension cross-classifies with categories in other dimensions and forms different subtypes like *form-I-triliteral-sound-active-participle*, *form-I-triliteral-sound-passive-participle*, *form-I-triliteral-sound-gerund*, etc. Not all these forms generate all types of verbal nouns—i.e. some of these forms do not have corresponding verbal nouns of all types. For example, locative nouns are generated from triliteral Form-I root verbs only. So for this type of verbal noun, classifying along other Forms does not generate any new type.
3.3 Construction Rule for Verbal Nouns

Before discussing the construction rules, we discuss a sample AVM for an active participle. After this, we will discuss other verbal nouns as well.

A sample AVM for an active participle is shown in Figure 6. All features of this AVM except SEM have been discussed before. SEM contains two features: INDEX and FRAMES. The INDEX feature registers reference to a discourse entity. FRAMES is the list of semantic frames which contains a frame for active participles which is the action frame. In this example the action frame is the write-fr which denotes write frame. This frame contains three indices: one for actor, another for the undergoer of the action (i.e. object) and the last one is for action or event.

We do not store this AVM as a lexical entry. Rather, this AVM is recognized by our lexical construction rules. The construction rule in Figure 7 shows how a verbal noun can be constructed from a verb. As we use the SBCG version of HPSG, the construction rule contains two parts: MTR which contains the AVM of the verbal noun and DTRS which contains the AVM of the base verb. This rule demonstrates how a Form—IA triliteral sound active participle is recognized from the lexeme of Form—IA triliteral sound root verb. The construction rule contains three placeholders for the three root letters. Thus from this construction rule, an active participle generated from letters ‘k’, ‘t’ and ‘b’ or ‘n’, ‘s’ and ‘r’ can be recognized. Note that there is no difference between constructing an active partici-
The construction of the active participle from Form–I verb is most regular. Other constructions are complex. For some verbs other forms even do not exist. Thus it requires further analysis.

Like that of the active participle, the construction of the passive participle from Form-I trilateral sound root verb is simple. There is just one pattern for this construction. So for all Form-I subtypes, the construction rule of figure 8 will be applicable. Derivation from other forms of verbs is complex and not regular. For some forms this type of participle does not exist either, which requires further analysis.

The verbs from which passive participles are derived should be transitive. For this reason, in the AVM of the DTR, the ARG-ST feature is not empty and its semantic index is co-indexed with the undergoer index in the event-fr. Note that the ARG-ST of the DTR contains one sign for object only, and it is in accusative case. It does not contain any sign for the actor. This is because, in Arabic, the actor is implicitly mentioned in the verb and the verb does not syntactically require the actor. If a subject is explicitly mentioned in the sentence, it can be parsed by phrasal construction rule.

Figure 7: Lexical rule for active participle construction
Figure 8: Lexical rule for passive participle construction
A locative noun can be generated from triliteral Form-I root verbs only. There are two patterns of derivation, and which pattern will be used for derivation is predictable. Form IA, IC, ID, IE and IG use the construction rule shown in Figure 9, whereas Form IB and IF use the construction rule shown in Figure 10.

In the AVM of a locative participle, we introduce a semantic frame \textit{locative-fr}. This frame has two features. These are the index for the event and the index for the location of the event. The event index of this frame is co-indexed with the event index of \textit{event-fr}. Thus it implements the location constraint of this participle.

![Figure 9: Lexical rule for the locative participle construction from Form IA sound root verb](image)

Figure 9 shows the construction rule for comparative participles. We have introduced a new semantic frame \textit{compare-fr} inspired by the analysis of Farkas, et.al. (Farkas and Kiss, 2000). This frame has three features. The first feature is “COMPARED”, which contains the index for the object that we want to compare. The second feature is “COMPAREWITH”. This feature contains the index for the object with which we want to compare. The last feature is the dimension of com-
Figure 10: Lexical rule for locative participle construction from Form IB sound root verb
This participle has an optional syntactic requirement, which is contained in the ARG-ST feature. The case of the required sign must be genitive. Its semantic index is co-indexed with the index of “COMPAREWITH” in compare-fr.

The construction of the remaining four types of verbal nouns is complex and we cannot resolve these by construction rules. We have to list the lexical entries for these verbal nouns individually. The reasons are discussed below.

Each verb form has a gerund that uses the most unpredictable pattern. Modeling its construction rule is a vast area of research. For now we can only list lexical
Hyperbolic Participles are generated only from triliteral sound Form-I root verbs. But not all verbs possess a corresponding hyperbolic participle. There are eleven patterns for deriving hyperbolic participles from verbs. However, we cannot predict from the root letters which of these eleven patterns will be used; neither can we infer the existence of a hyperbolic participle for the given root letter. So we have to list a lexical entry for each of these hyperbolic participles. Figure 12 shows a sample lexical entry for hyperbolic participle kattaabun which means the person who writes a lot. We have used the modifier-fr frame to capture the modification constraint.

Resembling Participles are similar to hyperbolic participles. They are generated only from triliteral sound FORM-I root verbs. There exists a large number of derivational patterns in this case. So, it is not feasible to formulate a lexical construction rule for these nouns. Thus in this case we also need to give the lexical entries. Figure 13 shows the lexical entry for katiibun which means a person who always writes. Like hyperbolic participle, here we have used the modifier-fr frame to capture the modification constraint.

Utilitarian Nouns are also generated from triliteral sound Form-I root verbs only. There are four patterns of derivation. For a given set of root letters it is unpredictable which pattern will be used. For this reason, despite the limited number of patterns, we have to list the lexical entries exaustively.
4 Conclusion

In this paper, we have captured the morphology of the Arabic verbal noun by extending the MORPH, SYN and SEM features. We have provided a detailed analysis of verbal nouns generated from triliteral sound Form I verbs. We have also devised inflectional rules which can be used to construct verbal nouns of different number and gender.

Immediate extensions of this work could be the modeling verbal noun from triliteral non-sound Form I verb and the analysis of verbal nouns based on quadriliteral verbs. An important aspect to note is that for some verb forms, there exists no specific construction rules, while for certain combination of root letters fixed construction patterns exist. Classifying these roots is an important research area not considered yet.

References


